Airflow pattern in a safe staircase

Influence of thermal convection

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1. Introduction

In recent history, it could be shown by means of hydrostatic considerations that differences in density between the ambient air and the air in the staircase ($\vartheta_{\text{Inside}}$ = 20 °C), especially in winter, lead to considerable pressure differences. On a cold winter's day ($\vartheta_{\text{outside}}$ = -10 °C), in the closed condition, the pressure difference between the environment and a 150 m high and 20 °C warm staircase is 200 Pa. Hence it appears that in case of such high pressure differences the doors leading from the relevant floor to the safe staircase cannot be opened securely. Furthermore, a discussion is taking place whether this static pressure difference will still be given when there is a convective flow through the staircase and consequently objections should be made to the method of operation with so-called smoke control pressurizing systems.

In this article we examine which convective flows due to the above static pressure difference are put into motion and which pressure differences appear between the safe staircase and the environment.

2. Modelling of the staircase

The staircase is considered to be a smooth chimney without stairs. The convective flow leads to a heat transfer within the walls. In this case, a stationary state is considered in the calculations.

In order to determine the influence of the individual component parts on the convective flow, first of all the theoretical border case of the frictionless motion is examined. Then the friction due to the roughness of the walls is considered. Finally, the individual resistances (bottom and top opening of the staircase) are observed. To begin with, the local resistances of the stairs are represented by an increased roughness of the staircase walls. If measured values are available on the pressure drop in such a staircase, then these can be used for the calculations instead.

Finally it is assumed that the staircase is impermeable to the environment. The so-called porous chimney (porous staircase and porous facade) will be dealt with in another article.

3. Mathematical interrelationship for convective flows in chimneys

When the staircase is closed, ideally the air temperature in the staircase is the same as in the rest of the building, thus 20 °C. The air density shall be ρ_{inside}. Near the building, the air density is $\rho_{outside}$. The hydrostatic pressure of the air against the height x is

$$p_{hyd}(x) = p_0 - \rho_{outside} \cdot g \cdot x \tag{1a}$$

On the contrary, the pressure in the staircase is

$$p(x) = p_0 - \rho_{inside} \cdot g \cdot x \tag{1b}$$

In both equations p_0 is the pressure near the ground (x=0).

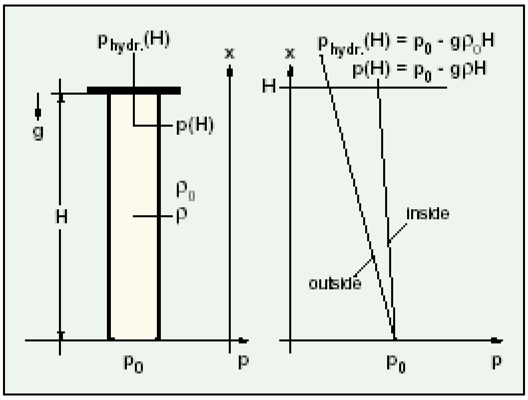

Fig. 1: Hydrostatic pressure distribution inside and outside the closed chimney [1]

Fig. 1 shows the hydrostatic pressure and pressure within the staircase in quality. Thus, the pressure difference at the head of the staircase (x = H) is

$$\Delta p_{hyd}(H) = (\rho_{outside} - \rho_{inside}) \times g \times H \tag{2}$$

The density is calculated for the isobaric atmosphere with the equation

$$\rho = \rho_0 \cdot \frac{T_0}{T} \tag{3}$$

where ρ_0 = 1,293 kg/m^3 and T_0 = 273 K (0 °C). The acceleration due to gravity is g = 9,81 m/s^2.

An upward force F acts on the lid at the staircase head in fig. 1, which can be calculated by using the relation

$$F = \Delta p(H) \cdot A \tag{4a}$$

where A is the cross-sectional area of the staircase. When removing the lid, according to the Newton's fundamental equation

$$F = m \cdot a \tag{4b}$$

the air mass within the staircase is accelerated. Thus the convective flow from the bottom to the top of the chimney is set into motion.

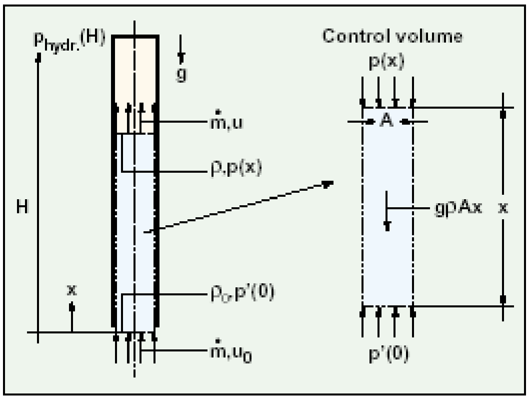

Fig. 2: Application of the theorem of momentum for stationary flow

If for the frictionless flow, the theorem of momentum is applied to the balance volume in fig. 2, hence it follows:

$$\dot{m} \cdot u - \dot{m} \cdot u_0 = \left(p'(0) - p(x)\right) \cdot A - g \cdot \rho \cdot A \cdot x \tag{5a}$$

If the equation for the conservation of mass

$$\dot{m} = \rho_{outside} \cdot A \cdot u_0 = \rho_{inside} \cdot A \cdot u \tag{5b}$$

and Bernoulli's equation for the frictionless flow at the inlet of the chimney

$$p_0 = p_0' + \frac{\rho_0}{2} \cdot u_0^2 \tag{5c}$$

are inserted into equation 5a and the difference in density between the environment and chimney is

$$\Delta\rho = \rho_{outside} - \rho_{inside} \tag{5d}$$

then the following equation is obtained for the pressure within the chimney:

$$p(x) = p_0 - g \cdot \rho_{outside} \cdot x - \left[\frac{\dot{m}^2}{\rho_{outside} \cdot A^2} \left(\left(\frac{\rho_{outside}}{\rho} - 1 \right) + \frac{1}{2} \right) - g \cdot \Delta\rho \cdot x \right] \tag{6a}$$

This equation can also be expressed as

$$p(x) = p_{hyd}(x) + \Delta p(x) \tag{6b}$$

where $p_{hyd}(x)$ is developed according to equation 1a and by comparing the equations 6a and 6b the following is valid for the pressure difference between the environment and chimney:

$$\Delta p(x) = -\left[\frac{\dot{m}^2}{\rho_0 \cdot A^2}\left(\left(\frac{\rho_0}{\rho} - 1\right) + \frac{1}{2}\right) - g \cdot \Delta\rho \cdot x\right] \tag{6c}$$

The unknown mass flow in equation 6c can be calculated from the so-called discharge condition. At the chimney head $p_{hyd}(H) = p\ (H)$, therefore $\Delta p(H) = 0$. Hence it follows:

$$\dot{m} = \sqrt{g \cdot \rho_{outside} \cdot A^2 \cdot H \cdot \frac{\Delta\rho}{\left(\frac{\rho_0}{\rho} - 1\right) + \frac{1}{2}}} \tag{7}$$

The equations 6 and 7 are only valid for the moment when the lid is opened. Contrary to a chimney where a source of heat maintains a permanent difference in density $\Delta\rho$ after opening the lid, air with a density $\rho_{outside}$ flows into the staircase. According to the laws of heat transfer, this air becomes slowly warmer within the chimney. The heat transfer within the staircase is then very similar to a continuously heated chimney.

For the frictionless stationary flow, Unger has derived in [1] contrary to equation 6c a more general form for the pressure difference between the chimney and environment. This is:

$$\Delta p(x) = -\left[\frac{\dot{m}^2}{\rho_0 \cdot A^2}\left(\left(\frac{1}{1 - \frac{\beta}{\dot{m} \cdot c_p} \cdot \int_0^x q(\xi) d\xi} - 1\right) + \frac{1}{2}\right) + \frac{g \cdot \rho_{outside} \cdot \beta}{\dot{m} \cdot c_p} \cdot \int_0^x \int_0^\eta q(\xi) d\xi d\eta\right] \tag{8a}$$

In the above equation, q(x) is the heating output related to the length and in dependency with the height x. ß is the temperature coefficient of dilation and can be expressed as:

$$\beta = \frac{1}{T_{chimneyhead}} \tag{8b}$$

The unknown mass flow in equation 8a can again be calculated from the discharge condition $\Delta p(H) = 0$. Hence it follows:

$$0 = -\left[\frac{\dot{m}^2}{\rho_0 \cdot A^2}\left(\left(\frac{1}{1 - \frac{\beta}{\dot{m} \cdot c_p} \cdot \int_0^H q(x) dx} - 1\right) + \frac{1}{2}\right) + \frac{g \cdot \rho_{outside} \cdot \beta}{\dot{m} \cdot c_p} \cdot \int_0^H \int_0^x q(\xi) d\xi dx\right] \tag{9a}$$

Both integrals in equation 9a can be expressed as:

$$\int_0^H q(x) dx = \dot{Q} \tag{9b}$$

$$\int_0^H \int_0^x q(\xi) d\xi dx = \dot{Q} \cdot \frac{H}{2} \cdot \Gamma \tag{9c}$$

According to Unger, Γ is a numerical factor which he calls shape factor. For both cases, the chimney heated at the base and the continuously heated chimney, the shape factors can be calculated by solving the double integral in equation 9c.

The chimney heated at the base is given by $q(x=0)=\dot{Q}$ and $q(x>0)=0$. If this is inserted into equation 9c, the shape factor for the chimney heated at the base is Γ = 2.

The continuously heated chimney is given by $q(x)=const=q_0$ with $q_0=\dot{Q}/H$. If this is inserted into equation 9c, the shape factor for the continuously heated chimney is Γ = 1.

The disadvantage of equation 9a is that the unknown mass flow cannot be solved explicitly. By using the first law of thermodynamics for the complete chimney, another equation can solve the problem. This is according to the first law:

$$\dot{Q}=\dot{m}\cdot c_p\cdot\left(\vartheta_{chimneyhead}-\vartheta_{outside}\right) \tag{10}$$

If equation 10 and 8b are inserted into equation 9a, the unknown mass flow is:

$$\dot{m}=\sqrt{\frac{1}{2}\cdot g\cdot\rho_{outside}^2\cdot A^2\cdot H\cdot\frac{\Theta}{\left(\frac{1}{1-\Theta}-1\right)+\frac{1}{2}}\cdot\Gamma} \tag{11a}$$

with

$$\Theta=\frac{\vartheta_{chimneyhead}-\vartheta_{outside}}{T_{chimneyhead}}=\frac{\vartheta_{chimneyhead}-\vartheta_{outside}}{\vartheta_{chimneyhead}+273K} \tag{11b}$$

For Γ = 2, equation 8a can be transferred into equation 6c and equation 11a into equation 7.

The only unknown value is then $\vartheta_{\text{chimneyhead}}$. It can be calculated from the temperature development of ducts, through which gases are flowing. According to [2]

$$\vartheta_{chimneyhead}=\vartheta_{building}+\left(\vartheta_{outside}-\vartheta_{building}\right)\cdot e^{-\kappa} \tag{12a}$$

with

$$\kappa=\frac{k\cdot U_i\cdot H}{\dot{m}\cdot c_p} \quad\text{and}\quad k=\frac{1}{\frac{1}{\alpha_i}+\frac{U_i}{2\cdot\pi\cdot\lambda}\cdot\ln\frac{U_a}{U_i}+\frac{U_i}{U_a}\cdot\frac{1}{\alpha_a}} \tag{12b}$$

In the above equation, U is the perimeter, λ the thermal conductivity of the staircase wall and α the coefficient of heat transfer. The subscript i relates to the staircase and the subscript a relates to the wall away from the staircase.

By using equation 11a, it is now possible to solve the mass flow explicitly, however the temperature at the chimney head is dependant on the mass flow according to equation 12a and 12b. Therefore the unknown mass flow can only be determined by iteration. For this, to begin with an estimated value is used for the mass flow (or for the volume flow which is multiplied by the relevant density) and then $\vartheta_{\text{chimneyhead}}$ is calculated with the equations 12a and 12b. Afterwards the mass flow is calculated by using the equations 11a and 11b. The new operand for the mass flow is the average value of the estimated value and calculated value. This iteration method is continued until the mass flow used in the equations 11a and 11 b corresponds to the calculated value. As this iteration converges very well, it can be used by hand without using a numerical program. However, one should use a spreadsheet program for these calculations.

The previous equations are only valid for the frictionless convective flow. In order to consider the friction, Unger has shown that in the equation for the pressure difference between chimney and environment (equation 8a) only the terms of friction have to be added. Considering the friction of a duct with the here given turbulent flow and according to [1] for the pressure difference between chimney and environment follows:

$$\Delta p(x) = -\left[\frac{\dot{m}^2}{\rho_0 \cdot A^2} \left(\left(\frac{1}{1 - \frac{\beta}{\dot{m} \cdot c_p} \cdot \int_0^x q(\xi) d\xi} - 1 \right) + \frac{1}{2} \right) + \frac{g \cdot \rho_{outside} \cdot \beta}{\dot{m} \cdot c_p} \cdot \int_0^x \int_0^\eta q(\xi) d\xi d\eta - K \cdot \dot{m}^2 \cdot x \right] \quad (13a)$$

with

$$K = \frac{8}{\pi^2} \cdot \frac{\lambda_t}{\rho_{outside}} \cdot \frac{1}{D^5} \quad (13\text{ b})$$

Equation 13b shows a general problem with the nomenclature. While in thermal engineering λ is referred to as the thermal conductivity, in fluid engineering λ is the frictional index for a duct. The subscript t stands for the turbulent flow. When the diameter is not round, the hydraulic diameter shall be used for diameter D in equation 13b.

Analogous to the method for the frictionless flow, the following relation for the mass flow results from the discharge condition and by inserting the first law (equation 10).

$$\dot{m} = \sqrt{\frac{\frac{1}{2} \cdot g \cdot \rho_{outside} \cdot \Theta \cdot H \cdot \Gamma}{\frac{1}{\rho_{outside} \cdot A^2} \cdot \left[\left(\frac{1}{1 - \Theta} - 1 \right) + \frac{1}{2} \right] + K \cdot H}} \quad (14)$$

Θ is again calculated according to equation 11b.

The mass flow is determined analogous to the frictionless flow, but has now to be calculated with equation 14 and not equation 11a.

If additional local resistances, e.g. bottom and top opening of the staircase, have to be considered, according to Unger [1] equation 13a has to be extended to the pressure drop of these local resistances. If $\Delta p_{\text{openings}}$ is the total pressure drop of all local resistances, then the pressure difference between chimney and environment can be expressed as:

$$\Delta p(x) = -\left[\frac{\dot{m}^2}{\rho_0 \cdot A^2} \left(\left(\frac{1}{1 - \frac{\beta}{\dot{m} \cdot c_p} \cdot \int\limits_0^x q(\xi) d\xi} - 1 \right) + \frac{1}{2} \right) + \frac{g \cdot \rho_{outside} \cdot \beta}{\dot{m} \cdot c_p} \cdot \int\limits_0^x \int\limits_0^\eta q(\xi) d\xi d\eta - K \cdot \dot{m}^2 \cdot x - \Delta p_{openings} \right] \tag{15}$$

The equation of mass flow is then:

$$\dot{m} = \sqrt{\frac{\frac{1}{2} \cdot g \cdot \rho_{outside} \cdot \Theta \cdot H \cdot \Gamma - \Delta p_{openings}}{\frac{1}{\rho_{outside} \cdot A^2} \cdot \left[\left(\frac{1}{1 - \Theta} - 1 \right) + \frac{1}{2} \right] + K \cdot H}} \tag{16}$$

Θ is calculated according to equation 11b and K according to equation 13b.

The iteration method for determining the mass flow again corresponds to the so far presented cases.

4. Flow conditions in an actual safe staircase

Hereinafter, an example of the flow situation is being examined, which has been taken from an actual construction project. The safe staircase has a height of 160 m and a cross-sectional area of 5,5 m * 2,5 m. The temperature within the closed staircase is the same as in the other parts of the building (20 °C). The outside temperature is -10 °C. Furthermore, the staircase wall is made of steel concrete [λ = 1,1 W/(m*K)] and is 24 cm thick. In order to consider the local resistances, the area of the opening at the bottom is 5 % of the cross-sectional area and at the staircase head 10 %.

In order to calculate the pressure difference between staircase head and environment, at first the density of the air has to be calculated at 20 °C and -10 °C. By using equation 3, you receive for T_{inside} = 293 K (20 °C) a density of ρ_{inside} = 1,20 kg/m^3 and for $T_{outside}$ = 263 K (-10 °C) a density of $\rho_{outside}$ = 1,34 kg/m^3. According to equation 2, the pressure difference between staircase head and environment is therefore $\Delta p_{hyd}(H)$ = 220 Pa. If the staircase is now completely opened at both the bottom and top, according to Newton's basic equation the enclosed air mass m is accelerated. In order to determine how long this starting process will take place, the velocity during this period can be considered as approximately constant. Thus the acceleration a can be expressed as

$$a = \frac{u}{t} \tag{17}$$

where u is the velocity of flow and t the time. For this however, the velocity of the convective flow has to be known. Therefore to begin with, the mass flow of the convective flow has to be determined.

According to equation 7 this is $\dot{m} = 298{,}553 \ \frac{\mathrm{kg}}{\mathrm{s}}$.

With the equation

$$u = \frac{\dot{m}}{\rho \cdot A} \tag{18}$$

the velocity u in steady state is 18 m/s. The mass of the steady air within the staircase is determined by the volume of the staircase multiplied by the density of air at 20 °C. Thus the mass m = 2640 kg. If equation 4a is equated with 4b und the acceleration a is replaced by equation 17, the unknown time is obtained.

With the previously determined values, the time necessary to accelerate the steady air within the staircase to the steady velocity is calculated at t = 16 s. During the acceleration process, the velocity is proportional to the time. I.e. the average velocity u_m is half the value of the steady velocity, hence u_m = 9 m/s.

If in equation 18 u is replaced by u_m and the mass flow is multiplied by the time for the starting process of t = 16 s, the air mass is 2376 kg.

The quantity of air handled during the starting process is therefore approximately equivalent to the air enclosed within the staircase. Already after 16 s, the 20 °C warm air approximately has escaped and cold outside air ingresses. Assuming that between the opening of the staircase (corresponds to the actuation of a smoke pressurizing system) and entering into the staircase more than approximately 16 s have passed, the pressure conditions during the starting process are of no importance. Then, as already described, cold air enters into the staircase, where it is heated. As mentioned in chapter 3, the heating of the air corresponds to the continuously heated chimney. Thus the so-called shape factor is $\Gamma = 1$.

The convective flow setting in can be calculated by iteration with the help of equation 8, 11 and 12 (frictionless flow), equation 13, 14 and 12 (frictional flow) or equation 15, 16 and 12 (frictional flow and additional local resistances).

In the absence of information on the pressure drop in staircases, the friction due to the wall and the flight of stairs is considered by using a friction index for a duct of $\lambda_t = 0{,}5$. Local resistances are located at the inlet and outlet openings. The drag coefficients are calculated by using the following equations [3]:

$$\varsigma_{chimneybase} = \left(1 - \frac{A_{opening}}{A}\right)^2 \tag{19}$$

$$\varsigma_{chimneyhead} = \left(1 + 0{,}707 \cdot \sqrt{1 - \frac{A_{opening}}{A}}\right)^2 \tag{20}$$

Fig. 3 shows for these cases the results of the pressure differences between staircase and environment (building) across the height. These results are valid for the stationary heat transfer within the staircase wall.

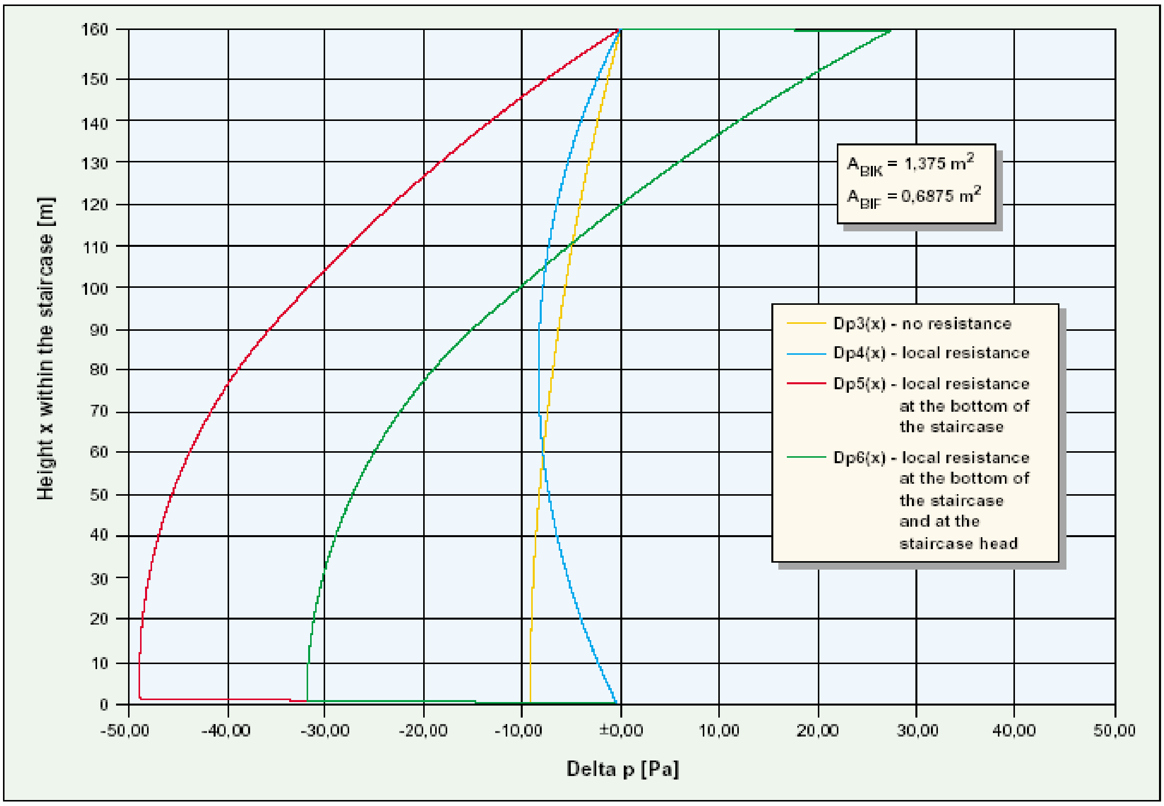

Fig. 3: Pressure difference between staircase and environment

Fig. 4 shows the convective flow rates relating to the outside air condition against the air temperatures at the staircase head.

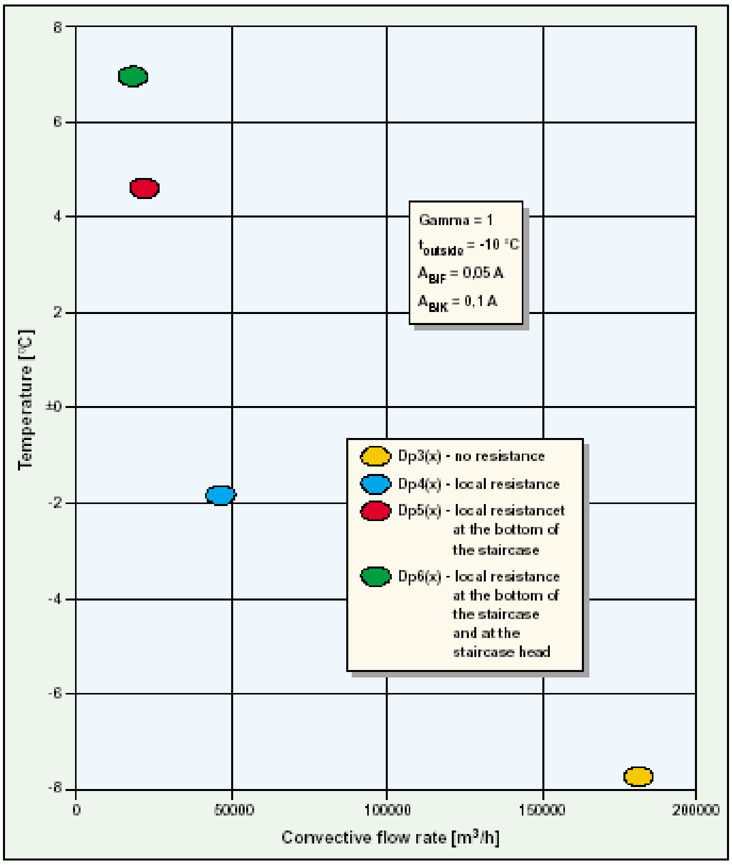

Fig. 4: Air temperatures at the staircase head

Fig. 5 shows for the chosen example existence of convective flows in dependency with changing outside temperatures.

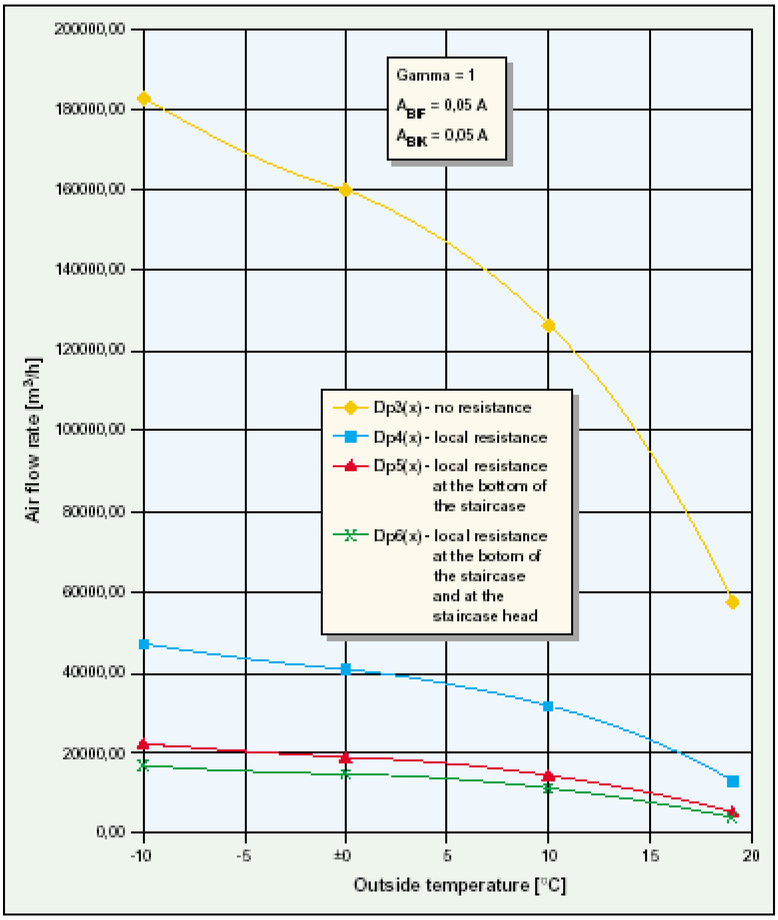

Fig. 5: Convective flow rates winter/summer

5. Conclusions

The calculations show that due to the flow through the staircase a negative pressure and overpressure compared with the environment (building) can be found. The amount of the pressure differences does not correspond to the static overpressure of the actual staircase that is closed. With the free convective flow setting in, the inadmissibly high pressure differences between staircase and building can be prevented. For this, all local resistances and the friction due to the wall and the flight of stairs have to be known and dimensioned in a proper manner.

The high airflow rates, which occur in case of the frictionless and frictional flow without additional local resistances (fig. 4), oppose to the experience with actual staircases. Hence it follows that the local resistances in actual staircases will decisively exercise an influence on the convective flow.

If one compares the pressure differences for one local resistance only at the bottom of the staircase with those for a local resistance at both the bottom and head of the staircase, it becomes clear that the resistor at the staircase head has a special function. With the help of this local resistance, the negative pressure differences in the lower region of the staircase are reduced. Whereas the pressure differences in the calculation examples with only one local resistance at the bottom of the staircase are very close to the maximum permissible 50 Pa, with the local resistance at the staircase head excessive pressure differences near the bottom of the staircase are prevented. After all, the air inlet and outlet openings determine whether free convective flows may occur or not.

Whereas for the closed staircase in winter the pressure force against the gravity force due to the difference in density will not create a hazard, the risk of inadmissible pressure differences at the crossings between doors and service units can only be prevented for the opened staircase by dimensioning the local resistances at the inlet and outlet and knowing the friction due to the wall and the flight of stairs. In winter however, the convective flow that sets in shall be considered when selecting the overpressure ventilators. As fig. 5 shows, even with low temperature differences between the environment and building (20 °C) it cannot be completely ignored.

6. Literature

[1] Unger, J.: Konvektionsströmungen; Stuttgart, B. G. Teubner Verlag 1988

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[3] Idelchik, I.E.: Handbook of Hydraulic Resistance, 2nd Ed., Washington, New York, Hemisphere Publishing Corporation 1986

7. Final remark

The authors hold that the largely unconsidered influences of the outer atmosphere on the function of the safety systems in the field of preventive fire safety can no longer be ignored. This article shall help to reduce the fear of having to deal with this subject and treating it mathematically, just because it is no longer general physics.

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